Electrostatic Fluctuations

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In the presence of the electrostatic fluctuation terms, the solution $\overline{f}_a$ of the drift kinetic equation is devided into the ensemble average and fluctuating parts:

$$\overline{f}_a = \left\langle \overline{f}_a \right\rangle_{ens} + \widehat{\overline{f}}_a$$

The lowest order solution in the drift ordering is given by the Maxwellian distribution function $\overline{f}_{a0} = \left\langle \overline{f}_{a0} \right\rangle_{ens} = f_{aM} \equiv \pi^{-3/2} n_a v_{Ta}^{-3} e^{-x_a^2}$ where $v_{Ta} = (2T_a/m_a)^{1/2}$ and $x_a = v/v_{Ta}$. To the next order, the ensemble-averaged drift kinetic equation is written as

$$v_\| \mathbf{n} \cdot \nabla \left\langle \overline{f}_{a1} \right\rangle_{ens} + \mathbf{v}_{da0} \cdot \nabla \overline{f}_{a0} + e_a v_\| E_\|^{(A)} \frac{\partial \overline{f}_{a0}}{\partial E_0}$$

$$= C_a(\left\langle \overline{f}_{a1} \right\rangle_{ens}) + \overline{\mathcal{D}}_a. \qquad (1)$$

In the right-hand side, we use the linearized collision operator and

$$\overline{\mathcal{D}}_a = -e_a v_\| \left\langle \widehat{E}_\| \frac{\partial \widehat{h}_a}{\partial E_0} \right\rangle_{ens}$$

where the nonadiabatic part $\widehat{h}_a$ is defined by $\widehat{\overline{f}}_{a1} = -\frac{e_a \widehat{\phi}}{T_a} f_{aM} + \widehat{h}_a$.

Using the Fourier representation in the fluctuating part of the drift kinetic equation, we have the nonadiabatic distribution as

$$\widehat{h}_{a\mathbf{k}} = \frac{\omega - \omega_E - \omega_{*a}^T}{\omega - \omega_E - \omega_{Da} - k_\| v_\| + i\nu_a} \frac{e_a \widehat{\phi}_\mathbf{k}}{T_a} f_{aM}$$

where $C_a(\widehat{h}_{a\mathbf{k}})$ is replaced with $\nu_a \widehat{h}_{a\mathbf{k}}$ and we used $\omega_E = \mathbf{k} \cdot \frac{c}{B} \mathbf{n} \times \nabla \Phi_0$, $\omega_{Da} = \mathbf{k} \cdot (\mathbf{v}_{a\nabla B} + \mathbf{v}_{a\mathrm{curv}})$, $\omega_{*a}^T = \omega_{*a} \left[1 + \eta_a \left(x_a^2 - \frac{3}{2}\right)\right]$, $\omega_{*a} = \mathbf{k} \cdot \frac{cT_a}{e_a B} \mathbf{n} \times \nabla \ln n_a$, and $\eta_a = d \ln T_a / d \ln n_a$.

The solution of the averaged drift kinetic equation for the plateau regime in the axisymmetric configuration is given by

$$\left\langle \overline{f}_{a1} \right\rangle_{ens} = \left\langle \overline{f}_a^{(l=1)} \right\rangle_{ens} - (\nu_a \mathcal{L}_a)^{-1} \langle \overline{\mathcal{D}}_a^{(l\geq 2)} \rangle + \overline{h}_a$$

$\left\langle \overline{f}_a^{(l=1)} \right\rangle = \frac{2v_\|}{v_{Ta}^2} \left[u_{\|a} + \frac{2}{5} \frac{q_{\|a}}{p_a} \left(x_a^2 - \frac{5}{2}\right)\right] \overline{f}_{a0}$, and the part contributing to the parallel viscosity by

$$\overline{h}_a = \epsilon \overline{\nu}_a^{-1/3} \left\{ \frac{x_a}{v_{Ta}} B \left[u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left(x_a^2 - \frac{5}{2}\right)\right] \overline{f}_{a0} \right.$$

$$\left. + \frac{1}{\nu_a} \int_0^1 \langle \overline{\mathcal{D}}_a^{(l\geq 2)} \rangle d\xi \right\} \int_0^\infty d\tau \sin(\theta - \overline{\nu}_a^{-1/3} \xi \tau) e^{-\tau^3/6}.$$

Here $\xi \equiv v_\|/v$ is the cosine of the pitch angle, $(\nu_a \mathcal{L}_a)^{-1}$ denotes the inverse of the pitch angle scattering operator, and $\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \rangle$ is the magnetic surface average of $\overline{\mathcal{D}}_a^{(l\geq 2)} = \overline{\mathcal{D}}_a - \frac{1}{2}\int_{-1}^1 \overline{\mathcal{D}}_a d\xi - \frac{3\xi}{2} \int_{-1}^1 \xi \overline{\mathcal{D}}_a d\xi$. Fluctuation effects on the solution are included through $\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \rangle$. The term $\overline{h}_a$ represents the distribution of the resonant particles ($|\xi| \ll 1$). The anisotropic distribution (or ξ-dependence) in the velocity space caused by the poloidal flows $u_{a\theta}, q_{a\theta}$ and by the fluctuation effect gives the source of the resonant particles which are responsible for the neoclassical and anomalous parallel viscosities. Then, noting that only $\overline{h}_a$ contributes to the flux averaged parallel viscosities, we obtain

$$\begin{bmatrix} \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle \\ \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\Theta}_a \rangle \end{bmatrix} = \begin{bmatrix} \left\langle \int d^3v m_a v_\|^2 \mathbf{B} \cdot \nabla \overline{h}_a \right\rangle \\ \left\langle \int d^3v m_a v_\|^2 \left(x_a^2 - \frac{5}{2}\right) \mathbf{B} \cdot \nabla \overline{h}_a \right\rangle \end{bmatrix}$$

$$= \frac{\sqrt{\pi}}{2} \epsilon^2 n_a m_a \omega_{Ta} B_0^2 \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} u_{a\theta} \\ \frac{2}{5}\frac{q_{a\theta}}{p_a} \end{bmatrix} + \begin{bmatrix} Y_{a1} \\ Y_{a2} \end{bmatrix}$$

$$= \frac{\sqrt{\pi}}{2} \epsilon^2 n_a m_a \omega_{Ta} B_0^2 \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} u_{a\theta} - W_{a1} \\ \frac{2}{5}\frac{q_{a\theta}}{p_a} - W_{a2} \end{bmatrix}$$

where Y_{aj} are the anomalous parallel viscosities due to the fluctuations and W_{aj} are the anomalous poloidal flow shifts W_{aj} given by

$$\begin{bmatrix} W_{a1} \\ W_{a2} \end{bmatrix} = \frac{v_{Ta} \tau_{aa}}{B_0} \int_0^\infty dx_a e^{-x_a^2} \frac{x_a^4}{\tau_{aa} \nu_a(x_a)}$$

$$\times \begin{bmatrix} \left(-\frac{3}{2} + \frac{1}{6} x_a^2\right) \\ \left(1 - \frac{1}{3} x_a^2\right) \end{bmatrix} \int_0^1 \frac{\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \rangle}{f_{aM}} d\xi.$$

References

[1] H. Sugama and W. Horton : to appear in Phys. Plasmas **2** (1995).